ISSN 2389-8186 E-ISSN 2389-8194

Vol. 4, No. 1 Marzo de 2017



Marzo de 2017 doi: http://dx.doi.org/10.16967/rpe.v4n1a2

ARTÍCULOS ORIGINALES

Demand forecasting system. Practical case of automated forecasting in catalog sales companies

pp. 23-39

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ABSTRACT This article presents the results of an academic work carried out in a manufacturing company of cosmetics which markets its products under the catalog or direct sale method. Thanks to it, it was possible to design an automated tool to forecast catalog product sales based on various marketing variables. The study was based on the detailed analysis of the causal variables driving and inhibiting the demand to later develop a demand forecast system software. This tool was created with the purpose of automating the processes involved in sales estimations or forecasts, which are processes based on statistics and mathematics through solid variables based on product, price, manufacturing and billing and soft variables corresponding to future strategic management plans such as marketing and sales.

The dynamics of the catalog retail companies require specific planning strategies and short reaction times that involve acquiring inventories for specific periods, which require more refined and automated forecast strategies.

KEYWORDS sales forecasts, regression, projection system, Bayesian techniques.

HISTORY OF THIS PAPER

The original version of this paper was written in Spanish. This English version is published in order to reach a wider audience. To cite this paper, please refer to its original version, as follows:

HOW TO CITE THIS ARTICLE?:

Boada, A.J. (2017). Sistema de proyección de la demanda. Caso práctico de predicción automatizada en empresas de venta por catálogo. *Perspectiva Empresarial*, 4(1), 23-41. http://dx.doi.org/10.16967/rpe. V4n1a2

RECIBIDO: 9 de agosto de 2016 **APROBADO:** 9 de diciembre de 2016

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ISSN 2389-8186 E-ISSN 2389-8194

Vol. 4, No. 1 Marzo de 2017 doi: http://dx.doi.org/10.16967/rpe.v4n1a2



Sistema de proyección de la demanda. Caso práctico de predicción automatizada en empresas de venta por catálogo

RESUMEN El presente artículo expone los resultados de un trabajo académico realizado en una empresa manufacturera de consumo masivo en el área cosmética, bajo el estilo de venta por catálogo o venta directa. A partir de este, se logró diseñar una herramienta automatizada para predecir productos por catálogo, en función de diversas variables de mercadotecnia. Dicho estudio se sustentó en un análisis detallado de las variables causales, impulsadoras e inhibidoras de la demanda, desarrollando posteriormente un software denominado sistema de proyección de la demanda, herramienta creada con la finalidad de automatizar los procesos inherentes a las estimaciones–predicciones de ventas, procesos fundamentados estadística y matemáticamente mediante variables sólidas en referencia al producto, precio, fabricación y facturación, y variables blandas correspondientes a planificaciones futuras de gerencias estratégicas como mercadeo y ventas.

La dinámica de las empresas de venta por catálogo impulsa a generar amplias estrategias de planificación futura, con tiempos cortos de reacción, que originan la adquisición de inventarios para periodos únicos, derivando así en estrategias de predicción más afinadas y automatizadas.

PALABRAS CLAVE predicciones de ventas, regresión, sistema de proyección, técnicas bayesianas.

Sistema de projeção da demanda. Caso prático de predição automatizada em empresas de venda de catálogo

RESUMO Este artigo apresenta os resultados de um trabalho acadêmico realizado em uma empresa manufatureira do setor de consumo massivo na indústria cosmética, sob o estilo de revenda de catálogo ou venda direta. Daqui estruturou-se uma ferramenta automatizada para visualizar produtos de catálogo em função de diferentes variáveis de marketing. Esse estudo se sustentou em uma análise detalhada das variáveis causais, impulsadoras e inibidoras da demanda, desenvolvendo depois um software chamado sistema de projeção da demanda, criado para automatizar os processos próprios das estimativas-predições de vendas, processos fundamentados estatística e matematicamente por meio de variáveis sólidas no que respeita ao produto, preço e fabricação e variáveis suaves no que respeita aos planejamentos futuros de gerências estratégicas como marketing e vendas.

A dinâmica das empresas de revenda de catálogo impulsa a gerar estratégias amplas de planejamento futuro, com tempos curtos de reação, que originam a aquisição de estoques para periodos únicos, acabando em estratégias de predição mais adequadas e automatizadas.

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HOW TO CITE THIS PAPER? ;CÓMO CITO EL ARTÍCULO?

CHICAGO:

Boada, Antonio José. 2017. "Sistema de proyección de la demanda. Caso práctico de predicción automatizada en empresas de venta por catálogo". *Perspectiva Empresarial* 4(1): 23-41. http:// dx.doi.org/10.16967/rpe. v4n1a2

MLA:

Boada, Antonio José. "Sistema de proyección de la demanda. Caso práctico de predicción automatizada en empresas de venta por catálogo". *Perspectiva Empresarial* 4.1 (2017): 23-41. Digital. http:// dx.doi.org/10.16967/rpe. v4n1a2

Introduction

Forecasts are vital for commercial organizations and for important administrative decisions, since the company's long-term plans are based on them. In the functional areas of finance and accounting, forecasts are the foundation for budget planning and cost control. The marketing department depends on sales forecasts to quantify its plans for new and regular products, sales strategies and the evaluation of the promotional impact to optimize decision making. The production and operations personnel use forecasts to make periodic decisions regarding process selection, capacity planning, the physical layout of the facilities and routine decisions about production plans, programs and inventories (Chase, Jacobs & Aquilano, 2005).

Currently, several multivariate statistical analysis techniques are used in the field of marketing and market research, such as strategic studies of market segmentation, correlation analysis, discriminant analysis, multidimensional scales and cluster analysis, among others (Prieto, 2013). However, Prieto's work focuses on the structural design of a statistical forecast model that could result in the consolidation of an automated tool in Visual Basic with an Access database that could allow the company to execute in real time future demand forecasts fed by causal marketing variables previously assessed according to the company's specific historical information.

Demand behavior in catalog retail companies differs significantly from traditional retail companies, since catalog retail companies control how they exhibit and promote their products in their catalogs, which have a specific valid time. Hence, the search for an automated application to be able to forecast the future demand for units in catalog retail companies has always been needed to be able to manage future demand efficiently in the areas of planning, logistics, purchasing and production.

This academic-industrial statistical project was carried out in order to analyze statistically the demand function of the products manufactured by a consumer goods company with direct sales (Avon Cosmetics) to understand the behavior of articles over time (during the validity of the sales catalogs) in their most detailed segmentation at the SKU (Stock Keeping Unit) level (Boada, 2013).

By developing a mathematical forecast model and based on the qualitative and quantitative variables previously registered from the planning stage, it was possible to design an automated system that allows maximum application of statistical theories according to certain market variables and to define a methodology to forecast sales demand by number of articles (even with offers). Thus, a forecast level that was consistent over time was proposed for the products of a direct sales company, which allowed the subsequent programming of an automated tool to execute projections and simulations in real time.

This automated tool is very valuable in emerging markets due to the potential increase of the market because of factors like growth rate of the group that uses the products (according to age or income), competition in both the formal and informal sectors and general environmental circumstances, such as the economic situation, consumer price index and lifestyle changes (Allen, 1985).

The lack of a stable demand is a common feature of emerging markets, so good forecasts become a key factor for the company's success. Wrong forecasts can lead to excessively large inventories, underestimation of prices or even loss of sales due to lack of inventories. Therefore, sales must be estimated in a fast and reliable manner, as it is very important for the planning of future activities that allow developing strategies to face a highly competitive market (Boada, 2011).

For this reason, the project resulted in a system based on products with similar characteristics that are offered over time in some catalogs, with the intention of having historical information on their line's behavior (Boada, 2013).

Marketing vs. management of future demand

In any consumer goods company, the marketing area establishes new strategies, releases and products based on creativity, new trends and business knowledge.

From this perspective, marketing strategies can be compared to strategies based on the most common competitive situations in any type of industry. Let us consider the red and the blue oceans (Chan-Kim, 1990):

• Blue oceans symbolize creative business ideas. They create and capture new demand, solve the value vs cost dilemma and align the

company's entire system with the purpose of achieving differentiation.

• In the red oceans, the traditional competitiveness perspective in market participation challenges competition, exploits the existing market demand and aligns the entire system based on strategic activities.

A forecast system for future demand such as DFS allows the company to focus on competitive strategies (especially applied to red oceans), providing greater freedom in line with the blue oceans.

Demand studies are the best way to know and define a company's marketing strategy. However, in the modern context of globalization, demand management has become the strategic link between creative areas like marketing and operational areas like logistics and supply chain.

The purpose of the departments or areas in charge of demand management and projection systems is mainly to quantify the creative strategies of the strategic departments in order to generate logistic planning of purchases, production and future inventories to guarantee product availability at a specific time. Likewise, assessing future demand provides significant benefits to the finance department and to general management in terms of income and future profit margins based on such creative scenarios.

A projection system establishes estimations based on the company's historical behavior (based on marketing strategies previously carried out). However, it is very important to state that macroeconomic situations, competitiveness and novel marketing strategies (blue oceans) generate new scenarios which are worth studying through additional analysis methodologies (Schiffman & Kanuk, 1991). They also create consumer perception until the actual purchase decision (including factors such as product, brand, store and payment choice), and even what happens after the purchase (analysis of periods of uncertainty, satisfaction, dissatisfaction or even repurchase).

Statistical forecast model: Is it possible to forecast future sale scenarios?

In order to face the marketing environment and make purchases, consumers experience a purchase decision process defined by Stanton (2007) as problem solving according to the various alternatives offered by the market. However, in mass planning in consumer goods companies the purpose of demand management is to coordinate and control all the demand sources, in order to be able to use the production system efficiently and deliver the product on time, achieving an optimal service level and respecting optimal cost levels (Chase et al., 2005).

Forecasts can be classified into four basic types: qualitative, time series analysis, causal relationships and simulation (Lind, 2015).

- Qualitative techniques are subjective. They are based on estimates and opinions.
- Time series analysis uses information related to past demand to forecast future demand, taking time as the main explanatory variable.
- On the other hand, causal forecast --mainly used in this study-- assumes that the demand is related to various underlying internal or external factors of the business environment. For this type of forecasts, the identification and assessment of causal variables are fundamental, as well as the use of marketing variables to forecast scenarios in companies with catalog-based sales like Avon Cosmetics, unlike retail companies.
- Finally, models based on simulation allow the person in charge of forecasting to assess and deal with various "assumptions" about the condition of the forecast. This aspect is mainly used in stock and financial asset valuation.

When using causal forecasts, the multiple linear regression method becomes fundamental, since it presents the main components of the demand for products or services: average demand, trend, seasonal and cyclical elements, random variation and autocorrelation (Chase et al., 2005).

Based on this structure, future scenarios were forecast by means of a composite predictive model that uses the benefits of a multiple regression model with the automatic adjustment of residuals provided by a Bayesian dynamic linear model. The basic structure was established as follows:

> = Result of Multiple Regression Model (First part) + Residual Adjustment Bayesian Dynamic Linear Model (Second part)

where the Bayesian model is obtained through historical information updated to t-1 time.

Ln (Demand)

First part: multiple regression model

It corresponds to the first part of the forecast model, that is, the creation of a multiple regression model based on market and marketing variables. Products with relevant history ("average life") were used. They were grouped into categories with similar characteristics and uses: deodorants, liquid colognes, powders, shampoos and conditioners, etc.

This is the fundamental part of the forecast, since it contains explanatory variables that behave according to product offers, advertising, shelf life and the number of sellers or representatives per campaign. For this reason, it is possible to know the impact of altering each market variable in the demand for the number of items for each product (Webster, 2000; Boada, 2012).

The multiple regression analysis is carried out in such a way that a dependent variable "Demand for articles of average life products" is related to two or more independent variables, Marketing Variables.

Demand for average life products	=	function (marketing variables) + errors
Y	= X.β	+ E

E(*Demand for average life products*) = *function (marketing variables*)

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$
(1)

If the values β_0 , β_1 , β_2 , ..., β_p were known, equation (1) would be used to calculate the average value of "y", given the values of the x_1 , x_2 , ..., x_n marketing variables. Unfortunately, these parameters are unknown and must be determined from the sample data. For this reason, point estimators of the previously described parameters were calculated.

$$\hat{Y} = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \dots + b_p \cdot x_p$$
(2)

where

 $b_0, b_1, b_2, ..., b_n$ are the estimators of $\beta_0, \beta_1, \beta_2, ..., \beta_p$ $\hat{y} =$ Estimated value of the demand for the number of items for average life products.

The method used to derive the estimated multiple regression equation is the "least squares method", which uses data from the sample to determine the b_0 , b_1 , b_2 , ..., b_n values that minimize the sum of the squares of the deviations between

the observed and estimated values of the dependent variable.

During the multiple regression analysis, each b_i regression coefficient represents an estimate of the variation in the demanded quantity of items for average life products when there is a unit change in an x_i marketing variable and when all other independent variables remain constant (Diebold, 1998).

The F-fisher and t-student tests were used to determine the validity of the parameters that go together with the independent variables through their estimators. Likewise, the assumption was made that the explanatory variables should be independent, since if there is a certain degree of correlation between them, there would be a problem of multicollinearity, which shows an inability to separate the individual effects of each variable on the quantity demanded for average life items (Dallas, 1998; Boada, 2011, 2013).

Marketing variables used during forecasting

Products with similar characteristics were grouped together and a regression model for the whole group such as liquid fragrances, shampoos and conditioners, deodorants, powders, etc. was calculated.

CAMPAIGN

It corresponds to the validity of the sales catalog. In the case of Avon Cosmetics, 18 campaigns are carried out annually and they are introduced in the system at the time of the estimation. When selecting a campaign, constant information related to the campaign can be automatically obtained, such as number of representatives or salespeople and inflation rate, among other aspects. This estimation process is carried out by campaign. However, estimates are assessed by product, estimating their competitiveness and internal "cannibalization".

ARTICLE CODE

Through the article code constant information related to the product can be linked and it can be obtained automatically through other internal systems of the company, for example: article name, category or family of products, full price and offer price established for each campaign, FIGURE 1. Demand forecast system. Module for introducing marketing variables

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Detalle de Articulo:			Variables Implicitas						
Código del Articulo:	63663		Precio:	1390		-			
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			Precio Base:	-	Statement of the local division of the local				
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			Tipo de Articulo:	12	Contraction of the local division of the loc	nce Origin 💌	Part Services		
							Procesar .	1	
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							Sale		

Source: Project results

type of article or weighting of the product within the same family, among other aspects.

NUMBER OF REPRESENTATIVES

It corresponds to the number of salespeople or representatives that the company has across the national territory for each campaign or sales catalog.

Advertising

It is a qualitative variable. It is determined by the location of the product's photo in the sales catalog. Some of the advertising indicators are: one page, spread and big product, among others. In spite of being a qualitative variable, its driving or inhibiting impact can be quantified according to the statistical technique of index numbers.

DISCOUNT

It is a qualitative variable that corresponds to the different promotional discounts designed for the article in each campaign. The characteristics of this variable are limited to the historical data of the products to be studied. If the company wants to offer a discount never offered before, the model will disregard the new activity, so that the estimator is in charge of linking them according to the existing ones. Some of the offers commonly used are "discount percentages", "both for", "packs", "two for the same or different", etc. Despite being a qualitative variable, it is possible to quantify its driving or inhibiting impact according to the statistical technique of index numbers.

During the development of this system, it was determined that for a better study of the articles it was necessary to take into account certain indicators that are not specified in the original application of the client company. Therefore, there was a need to review and manually restructure all the data provided, creating a new detailed scheme of advertising and promotional indicators that would result in an optimal and robust structure of continuous registration of information in real time, similar to a big data system for estimation and simulation of real-time scenarios.

Price

Sale price as an absolute variable is a mixture of other indicators, such as passage of time, the country's historical inflation, the variation of production costs, among other aspects. The system takes into consideration the following characteristics:

- Sales price-inflation rate ratio. The sale price of every product must have an increase according to the country's inflation rate (per sales campaign). If the price increases less than proportionally with respect to the inflationary price, the quantity demanded may increase without the need to make any offers. The same would happen during campaigns in which the price remains constant, while inflation continues its course.
- The presence of a relationship between the discounted price and the normal price (without discount) is necessary, since it would indicate the additional weight that the quantity demanded under that promotional indicator would suffer.

CATEGORY OR FAMILY OF PRODUCTS

It represents a group of products whose physical or use characteristics are similar. This grouping is ideal to identify behavioral patterns and therefore achieve more effective forecasts; for example, groups of deodorants, shampoos, lotions, among others.

TYPE OF ARTICLE

By means of this variable, the approximate "weight" of each product with respect to a reference article is quantified. Although the models group similar products, the system forecasts individually by article because the sales level is clearly individual and independent. It is possible to add new articles to the system, as long as they belong to those that have an elaborated model by previously determining the possible average demand per campaign.

This variable is used as initial forecast information, since later the forecast system will automatically adjust its sales level according to the latest campaigns.

DIFFERENT SIZE

Through this variable, the forecast system considers the possible variation of the demand if the product is offered in different sizes in any given campaign. It is currently available for liquid colognes and shampoos-conditioners. For example, 100 ml and 15 ml cologne bottles with the same fragrance.

TYPE OF ESTIMATES

This is a subjective variable, as it is determined by the estimator's judgement. In this way, the system can make controlled under or over estimates. It has three overestimation levels and two underestimation levels.

This feature is very important, because it allows the systematic performance of the estimator if an unexpected situation arises in the system (without having to wait for the model to be updated). For example: extreme changes in the country's situation, special contingencies and novel offers.

The following offers can be highlighted:

- a) when a product is given for free with the purchase of a cheaper product;
- b) when two or three items are offered at a lower price than the individual item, etc.

The project mainly used the S-Plus 4.5 software. Additionally, SPSS 7.5 and Microsoft Excel, among others, were used.

How to quantify qualitative variables

For this project, a calculation procedure was established in order to weigh the driving or inhibiting impact of each term of a qualitative variable on the demand behavior of a family of products.

For these variables it was necessary to create a quantification level measuring the impact of each characteristic within each product family. For this reason, the weighting procedure through simple index numbers was used. It uses a scale transformation that manipulates the values to ensure compatibility with other scales (Webster, 2000). The components for calculating the indices were:

PDA _{Absence}: Basic Product Demand Average when there is absence of the term of the qualitative variable calculated for each family of specific products. It corresponds to the most basic and common indicator of the qualitative variable of a product belonging to any family in the catalog.

 Σ (Demanded amout of items when there is absence belonging to each family)

PDA_{Absence}:

Times when there were products with absense of the qualitative var for that family AD _{Characteristic (i):} Average Demand calculated for each family of specific products according to each characteristic of the qualitative variable (i) registered in its historical sales.

4.0	Σ (Demanded amount of items in the qualitative var "i" belonging to each family)
AD _{Characteristic (i)} :	Times when there were productswith the qualitative vor "i"for that family

Then, the qualitative impact indicators (QII) are calculated for each average demand (AD) by comparing their percent difference with the basic product demand average (PDA). This is smoothed logarithmically to maintain the linear stability of the original semi - log model.

In this sense, the driving or inhibiting qualitative impact indicator (QII) for each "i" characteristic of each "j" qualitative variable would be calculated as follows:

$$QIC_{(i,j)} = \left(\frac{In \left(AD_{\text{Characteristic}(i,j)}\right)}{In \left(PDA_{\text{Absence}(j)}\right)} - 1\right) \cdot 100$$
(3)

In this way, there were as many indicators of a qualitative variable as characteristics that were presented in the product family's historical data.

Similarly, if a discount that is not included in the discount chart becomes repetitive in the products studied, a master user can calculate its effect and introduce it in the list of indicators, so that it can be used later by the estimators.

Example of the multiple regression statistical model (MRM) developed

Taking into consideration these input variables, it was possible to perform several tests on product families of Avon Cosmetics. Interesting results were obtained in terms of adjustment, consistency and robustness by using the method for variable inclusion called "backward" or descending elimination. In the studies conducted, R2 adjustment levels were between 70% and 85% with minimal multicollinearity among variables and a statistically acceptable behavior of residuals in terms of randomness and normality and with broad variance stability (Boada, 2013).

The results obtained for one of the families analyzed for this multinational company are presented below.

FAMILY: WOMEN'S LOTIONS

Definition: All liquid and creamy hand and body lotions to be used by women.

A run was made through the S-plus® statistical application to determine the regression equation in the women's lotions product family. The results obtained (at the level of β coefficients) showed a significant impact on the logarithmic transformation of the demand by SKU in each sales campaign, as shown in Table 1.

The multivariate regression equation for this specific family with the information processed through the S-Plus® statistical application was:

$$Ln(Demand) = \begin{cases} 453,89170000 - 0,13596080 \\ * V18 + 0,03887495 * V22 + \\ 1,22522600 * V25 + 0,00046793 \\ * V26 + 0,00132950 * V27 + \\ 0,00842663 * V28 - 0,2242180* \\ V30 \end{cases}$$
(4)

With this multivariate regression formula, the initial forecast of the statistical model can be established based on the enhancing and inhibiting impact of the marketing variables.

TABLE 1. Estimated coefficients for the women's lotions product family

VARIABLE	CODE	COEFFICIENT	INFLUENCE ON LN (DEMAND)
Constant		453,89170000	
Different size binary variable	V18	-0,13596080	Inhibiting
Type of item indicator	V22	0,03887495	Enhancing
Price indicator	V25	1,22522600	Enhancing
Exposure variable	V26	0,00046793	Enhancing
Discount variable	V27	0,00132950	Enhancing
Promotional cannibalization variable	V28	0,00842663	Enhancing
Date variable	V30	-0,22421800	Inhibiting

Source: Project results

When verifying both the "F" statistic and the "p" values resulting from the "t" tests, it was seen that all the estimated coefficients for the variables taken into consideration are significantly different from zero with an 81.69% R2 coefficient of multiple determination. This indicates a high percentage of variability, which is explained by the model with over 1,500 data.

COEFFICIE	NTS:			
	VALUE STD.	ERROR	T VALUE	PR(> T)
(Intercept)	453.8917	48.7160	9.3171	0.0000
V18	-0.1360	0.0673	-2.0211	0.0435
V22	0.0389	0.0009	41.7599	0.0000
V25	1.2252	0.1032	11.8731	0.0000
V26	0.0005	0.0002	2.3624	0.0183
V27	0.0013	0.0000	29.7885	0.0000
V28	0.0084	0.0009	9.5151	0.0000
V30	-0.2242	0.0243	-9.2431	0.0000

Residual standard error: 0,7114 on 1099 degrees of freedom

Multiple R-Squared: 0,8169

F-statistic: 700,3 on 7 and 1099 degrees of freedom, the p-value is o

FIGURE 2. MRM residuals graphics for women's lotions

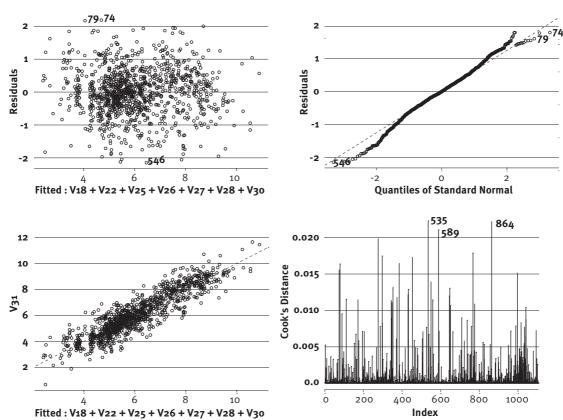
Likewise, residual analysis demonstrates normality and randomness.

Second part: Bayesian model

The estimation of the results obtained through multivariate regression models is based on the use of marketing variables according to marketing strategies. However, according to Peter (2006), it is possible that the consequent behavior of product demand (in regards to its tendency) changes over time in the case of scenarios like:

- A new competitor in terms of product or company.
- Variation in the number and length of sales campaigns.
- Extreme changes in the country's situation.
- New sales level because of product redesign.
- Excessive changes to sale price.

Bayesian statistics provide an ideal theoretical framework for data modeling, since problems can be treated with axiomatic clarity and flexibility



Source: Project results

simultaneously, which makes it possible to develop coherent inferences in order to establish a robust, stable and consistent procedure that facilitates programming solidification in a demand forecast system (DFS).

This Bayesian part is mostly executed, given the limitations of the multiple regression model, in terms of item demand, which remains constant in regression models (Pericchi, 2002). These causes can produce an alteration in the demand level according to the type of article. Through the dynamic Bayesian model, the effect produced by any of these causes can be determined while multiple regression model do not allow this. This can be done with a few catalogs (campaigns) of historic sales in order to adjust the evolution of the amount demanded over time (West & Harrison, 1989). In the same way, it takes into consideration continuous subtle changes, that is, it can adapt automatically to slight modifications in sales levels (Boada, 2011).

Although these multiple regression models are applied in categories (families) with similar characteristics (deodorants, shampoos, liquid colognes, etc.), their adjustments are made individually by product immediately after obtaining information on their actual demand. For this reason, their reaction and adjustment capacity is superior to that of any estimator.

First-order dynamic linear model

This model corresponds to the second stage of the forecast statistical model and it is applied directly on the residuals previously specified to obtain a forecast distribution that will evolve over time and whose levels will be added to the result of a future "t + 1"forecast provided by the multiple regression model.

THEOREM 1

For each point in time t = 1, 2, 3, ..., let us consider the following observation and system equations (West & Harrison, 1989; Pericchi, 2002):

Observation equation

 $y_t = \mu_t + \upsilon_t$, where $\upsilon_t \sim N(0, V_t)$

System equation

 $\mu_t = \mu_{t-1} + \omega_t$, where $\omega_t \sim N(0, W_t)$

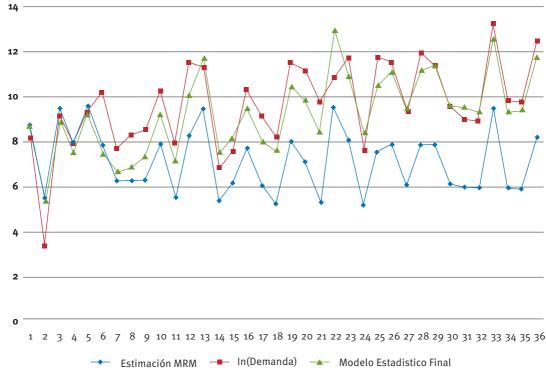


FIGURE 3. Forecast chart created by the demand forecast system (DFS) for a specific Avon Cosmetics product

Taking $\Pi(\mu_0 | D_0) \sim N(m_0, C_0)$ as initial information, where the assumption of randomness and normality of residuals of the multiple regression model will be maintained, $m_0 = 0$, y $C_0 = 1$ (Boada, 2002).

This Bayesian component generates a forecast which is adjusted for each new level of demand for each item. It is calculated through a priori learning according to the errors between the forecast of the multiple regression model and the logarithm of the demand (after the sale has occurred).

This new level of demand for each item creates an automatically adjustable level at "t", which will maintain the consistency of the multiple regression model in products that increase or decrease their demand level as they evolve over time. Likewise, in case the level of demand of a certain product begins to gradually increase (or decrease) in the latest catalogs as a product of macroeconomic situations or competitiveness that was not taken into consideration in the original regression model, there is disparity between the behavior of real demand and the forecasts of the multiple regression model.

Automated application: demand forecast system (DFS)

The research project began in 2000 thanks to the cooperation between Universidad Simon Bolivar and Avon Cosmetics of Venezuela, in the search of a standardized procedure that would forecast future sales scenarios based on the company's historical behavior. An important part of this agreement was the development of a statistical demand forecast system called DFS, an automated application designed to provide an additional tool to the demand planner. It was also sought that this application, together with the planner's own experience, would allow him to obtain with less effort a greater level of certainty when forecasting the quantities demanded for each product. Originally, the project was conceived according to

FIGURE 4. Demand forecast system. Historical information search

alle del Producto	Send and a stream of	and a start start		- 11.000		and the second				
etalle de Articulo:			Historico del	Articulo:						
Código del Articulo:	63663		Año	Campaña	Demanada	UPR	Pror	noción	Publicidad	1.
	Contraction of the second s		2002	8	8937		7 Ninguna oferta		Spread (ubicados en las dos pa	
	COOL CONFID ANTIP.F	OOL ON HED.	2002	9	7645		4 Ninguna oferta		Ubicación en una pagina (lo qu	
Categoria:	1 DESODORANTE	· ·	2002	10	13542				Ubicación en una pagina (lo qu	
			2002	11	89520			to (de 30% en ad		100
	The second s		2002	12	5513		Ninguna oferta		Ubicación en una pagina (lo qu	
nformación Relevan	le	March March 1997	2002	13	5487		3 Ninguna oferta		Ubicación en una pagina (lo qu	
Precio:	1390	and the second second	2002	14	788				Big Product (solo para desodora	
Año:	2003		2002	14	70799				Big Product (solo para desodora	
Campaña:	Children and and and and and and and and and an	Contraction of the	2002	15	6918 9208		8 Ninguna oferta 8 Ninguna oferta		Ubicación en una pagina (lo qu	
	01		2002	16	3208		 Ninguna oferta Ninguna oferta 		Ubicación en una pagina (lo qu	
U.P.R	0,108247311828		2002	18	25536				Ubicación en una pagina (lo que Big Product (solo para desodore	
Representantes en Campaña	93000		1 Deservation	A DECISION OF A DECISION	23036	0.235042652	c A de descueri	io (nasta el 20%)	big Product (solo para desodora	9
en comporto			Estimado del		1.10				Constant States of States	
D	emanda Real: 10067	Contraction of the	Año 2003	Campaña	Durinter	Promoción		115	Publicidad	
Estimado Model	o Estadístico: 10147		2003	2	and the second se	Jual en "Paque			una pagina (lo que ocurre norm-	
stimado Final del D			2003	3	Ninguna ofer		dos productos igi		una pagina (lo que ocurre norm- una pagina (lo que ocurre norm-	
. Thinddo T mor der e	-cpartamento. [8500		2003	4			adalanta) Colo		una pagina (lo que ocurre norm-	
			2003	5	Ninguna ofer		radelarite), Solo		una pagina (lo que ocurre norm-	
Predicción Mínima	del Modelo: 7586		2003	6	Ninguna ofer				olo para desodorantes Cuidado	
Predicción Máxima	del Modelo: 13573	Contraction of the	2003	7		nto (hasta el 2	521		una pagina (lo que ocurre norm-	
	the second s		2003	8	Ninguna ofer			and the second se	una pagina (lo que ocurre norm-	
Desviación de la E		1 2	2003	9	Ninguna ofer				una pagina (lo que ocurre norm-	
Modelo vs Dem	anda Real	Statement of the second	2003	10		nto (hasta el 2	5%)		una pagina (lo que ocurre norm-	
Desviación de la E vs Deman		-15 %								
				Promo	ciónes Ante	riores				
					Año (Campaña	Demanda	UPR	Publicidad	
				0.20 2 200	2002	5			Jbicación en una pagina (lo q	
					2002	6			Ubicación en una pagina (lo q	
				235 2 2	2002	8			pread (ubicados en las dos p	
				State of the	2002	9			Jbicación en una pagina (lo q	
				STATE OF	2002	12			Jbicación en una pagina (lo q Jbicación en una pagina (lo g	
					2002	15			Jbicación en una pagina (lo q	
					2002	16			Jbicación en una pagina (lo g	
				COLUMN DO NOT	2002	17			Jbicación en una pagina (lo g	

AVON COSMETICS DE VENEZUELA

Source: Project results.

the particular requirements of direct sales companies and with specific characteristics of each country's Avon Cosmetics. However, the pattern discovered can be extended to similar companies by creating statistical forecast models adjusted to the information handled.

The backbone of the demand forecast system is a statistical prediction model developed by specialists in the area, based on the multiple regression methodology with market variables and the automatic updating technique of the Bayesian dynamic linear model.

Through the automated application, the product's history, its discounts (offers) and advertisement (location in the catalog) by campaign are shown. In addition, it has an automatic option for filtering information according to discount, where the estimator can see the campaigns (demand and UPR) where the article had the same type of offer. It shows the error percentage (by excess and defect) of both the statistical model and the estimates department (Boada, 2000).

In addition to proposing an estimate, the statistical forecast model establishes a forecast interval, in which the estimator can input final estimates through tolerance bands (confidence intervals). These intervals are adaptable according to the accumulated error between the statistical model and the final real demand. If the error is low, the amplitude of the intervals will be reduced.

Estimated and current costs of each product per campaign can be displayed, as well as their gross profit.

One of the drawbacks found had to do with products with slight variations in their type of packaging or label, which leads to a code change and, therefore, a total loss of their sales history. To counteract this, a module was created to introduce a redesign and link the new code with the old one, thus maintaining the product's history.

Similarly, when a new product that cannot be coupled directly to another one is launched, there is an option called "product start", where the estimator inputs the expected average number of items to be sold per campaign and the system is adjusted to start the forecast based on that premise. If the product shows another sales level, the model will be adjusted automatically when incorporating the product's actual demand.

FIGURE 5. Demand forecast system. Costs and gross profit search

	And the second se					CHARLES CO. 19 (1)	
H	listorico del A	rticulo:					
	Año	Campaña	1 Demanada	UPR	Promo	ción	Publicidad
2016	2002	8	8937	0.092627717	Ninguna oferta		Spread (ubicados en las dos pa
1883	2002	9	7645	0.081464984	Ninguna oferta		Ubicación en una pagina (lo qu
IS NOT	2002	10	13542	0,140394165	% de descuento	(hasta el 25%)	Ubicación en una pagina (lo qu
	2002	11	89520	0,884261683	% de descuento	(de 30% en ad	Super Oferta (Generalmente ub
231131	2002		5513	0,056511711	Ninguna oferta		Ubicación en una pagina (lo qu
	2002	13	5487	0.054897449	Ninguna oferta		Ubicación en una pagina (lo qu
	2002	14	788	0,0079294	Paquetes (Tres a	artículos fijos p	Big Product (solo para desodora
100	2002					erentes II (dos	Big Product (solo para desodora
	2002					12.00	Ubicación en una pagina (lo qu
3113							Ubicación en una pagina (lo qu
1923			and the second sec				Ubicación en una pagina (lo qu
1981	2002	18	25536	0,235042892	% de descuento	(hasta el 25%)	Big Product (solo para desodora
E	stimado del A	rticulo:					
	Año	Campaña	1				Publicidad
	2003	2	Precio Individ	ual en 'Paquel	tes"	Ubicación en l	una pagina (lo que ocurre norm-
	2003	2	Dos iguales o	diferentes II (d	los productos igua	Ubicación en l	una pagina (lo que ocurre norm-)
1995	2003	3					una pagina (lo que ocurre norm-
	2003	4					
11111							una pagina (lo que ocurre norm-
11181							olo para desodorantes Cuidado
							una pagina (lo que ocurre norm-
							una pagina (lo que ocurre norm-
							una pagina (lo que ocurre norm-
	2003				-)	oocacion en	ane pegine to que costre nomi
MI	1.0720204420	-20	Costo				
		20	- 1 C C C C				
		-		Contraction of the		- Andrews	
AL	0,2701562118	86748		Losto Ac	ual: 250,39	E STORE	
	-7,9412543000	00003E-03		Costo Estim	ado: 259		
	2,7015621188	6748			P.: 70.02 %	-	
					and the second second	State of the local division of the	
	13,701562119						Salir
		Año 2002 2002 2002 2002 2002 2002 2002 2002 2002 2002 2002 2002 2002 2002 2002 2003	2002 8 2002 9 2002 10 2002 11 2002 12 2002 13 2002 14 2002 14 2002 14 2002 14 2002 14 2002 14 2002 14 2002 15 2002 16 2002 16 2003 16 2003 2 2003 2 2003 2 2003 3 2003 4 2003 5 2003 6 2003 7 2003 9 2003 9 2003 9 2003 9 2003 10	Año Cançaño Demanado 2002 8 8377 2002 9 7645 2002 10 13542 2002 10 13542 2002 11 89520 2002 12 5513 2002 13 5487 2002 14 7093 2002 15 5318 2002 15 5318 2002 16 5038 2002 17 10220 2002 18 25536 Estimado del Atticulo: Mario Arroya 2003 2 Dos iguales o 2003 2 Dos iguales o 2003 3 Ningans det 2003 6 Ningans det 2003 10 4 de descue 2003 10	Año Cançaña Demanada UPR 2002 8 8337 0.03652777 2002 9 7445 0.06146394 2002 10 13542 0.04125194 2002 11 95500 0.04265163 2002 12 5513 0.06551711 2002 13 5487 0.0795294 2002 14 70793 0.71242423 2002 15 6518 0.00795294 2002 15 6518 0.0795294 2002 15 6518 0.0795294 2002 15 6518 0.0795294 2002 15 6518 0.0795294 2002 15 6518 0.0795294 2002 17 10220 0.01043329 2002 17 12200 1.01043329 2003 2 Doci guaderio diretal effection dividual en 'Paquet odividual en 'Paquet odidual d	Año Canpaña Demanda UPR Phono 2002 8 8837 0.03827717 Ningan ofetta 2002 9 7645 0.081456944 Ningan ofetta 2002 10 13542 0.084251693 X de descuerto 2002 11 9550 0.084251693 X de descuerto 2002 12 5513 0.069511711 Ningan ofetta 2002 13 5487 0.0759234 Propater of 2002 14 70739 0.712428425 Doe guater of 2002 15 6318 0.0795234 Propater (Treixid) 2002 16 9308 0.09310705 Ningan ofetta 2002 17 1220 10.0132297 Ningan ofetta 2003 2	Año Cançaña Demanda UPR Promocin 2002 8 8377 0.09362777 Ninguna detta 2002 10 1542 0.09362777 Ninguna detta 2002 10 1542 0.08164549 Ninguna detta 2002 11 19520 0.084251603 X. de descuento (basta el 25%) 2002 12 9513 0.06511711 Ninguna detta 2002 14 7073 0.712428423 Dora guatero differentes II (dora finguna detta 2002 15 6318 0.00752319 Ninguna detta 10043239 2002 15 6318 0.00752319 Ninguna detta 1004329 2002 16 9208 0.0752419 Ninguna detta 10555111 2002 17 10220 0.10432929 X. de descuento (hasta el 255) Ubbaccin en 2003 2 Dora guatero diferentes II (dor productora igu- Ubbaccin en 2003 4 X. de descuento (hasta el 255) Ubbaccin en 2003 2

34

RPE

FIGURE 6. Demand forecast system. Module to link old and new codes of the same product

Pro	ducto Viej	0		Producto Nuevo				
Código: (9537) Campaña: (13		18	Código:	20700				
		Second I	Campañ	a: 15				
A	Año: 1996			Año:	1996			
Pre	oductos Re Código A	200 C	dos Campaña Fin	Año Fin	Código Nuevo	Car		
-	95378	ancenor	13	1996	20700	15		
-	27770	1	12	2001	27771	13		
	63136		04	2003	63843	06		

Source: Project results.

FIGURE 7. Demand forecast system. New product start module

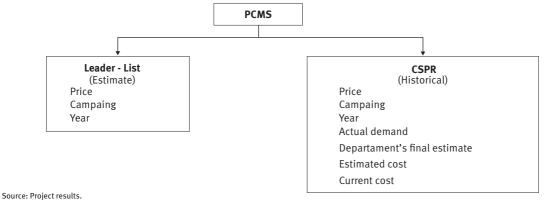
One of the main challenges of the automated application was the direct interface that the DFS must have with marketing information systems (MIS). The interface consolidates the set of procedures and methods for the collection, analysis and presentation of planned and periodic information for subsequent use in marketing decisions in all its stages (Prieto, 2013). It provides the information at figure 8.

Forecasts are made based on an estimated inflation rate and number of salespeople. However, the automated application updates this information in real time in order to maintain direct communication between all associated departments. The forecast system makes direct copies of the company's primary systems because it is a tool for simulating possible future scenarios based on the plans previously made by the strategic marketing and sales departments. The Leader List and

Da	tos de In	icio del Pr	oducto Nuevo					
Có	digo Pro	ducto: 58	412 Camp	aña: 2003	3/04	Categoria: 1	DESODORANTE	•
Гa	sa de Inf	lación: 1	Prec	io: 1256		nedio Estimado a Demanda:	526	Procesar
P10	ductos I	niciados						
15	Código	Campaña	Tasa Inflación	Fecha	Precio Base	MT	FT	CT
	63983	15	1,59	399,77777	2995	0,144219403	0	1,666666667
83	63974	2	1,69	300,05555	3085	0,090844765	0	1,666666667
	63987	16	1,59	399,83333	2995	0,077606354	0	1,666666667
1	63979	17	1,48	399,88888	2995	0,035272171	0	1,666666667
	63996	1	1,67	2000	3085	0,064051971	0	1,666666667
50	65452	16	2,25	302,83333	4500	0,159930452	0	1,666666667
202	63094	12	1,03	300,61111	2549	0,047101193	0	1,666666667
20	63288	15	1,72	300,77777	3085	0,293430064	0	1,666666667
	63382	4	2,71	398,16666	1869	0,072451417	0	1,666666667
_	96120	1	6.01	1996	903	0.01355/316	0	1 666666667

Source: Project results.

FIGURE 8. Flowchart of the demand forecast system with Avon Cosmetics' corporate software



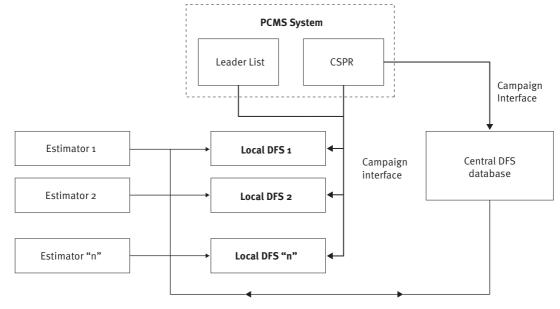


FIGURE 9. Work, communication and interface flowchart with the demand forecast system

Source: Project results.

PCMS systems correspond to Avon Cosmetics of Venezuela's marketing information systems (MIS).

This specific linking procedure generates different routes in the databases (estimated and historical), producing great adaptability between the demand forecast system and any other master information system of the company. In addition, it has a short formulation, calculation, search and storage time for the data processed.

Likewise, the system can estimate the products with double or triple offers because it analyzes the prices provided by Leader List. In this way, an item can be forecast both in terms of offers and individual prices in the same sales campaign. For these cases, the ID is determined by sales code + sales campaign + product price.

Although the statistical model is continuously and automatically updated, it is possible to make forecasts regardless of the loaded campaigns. The difference is that the estimate is more accurate if the estimated campaign is approaching (the preliminary estimates will be updated automatically as the review and final stage approach). In the same way, subsequent adjustments can be made to the estimates under operational versions, thus observing the modification of the estimates according to the results obtained in trends.

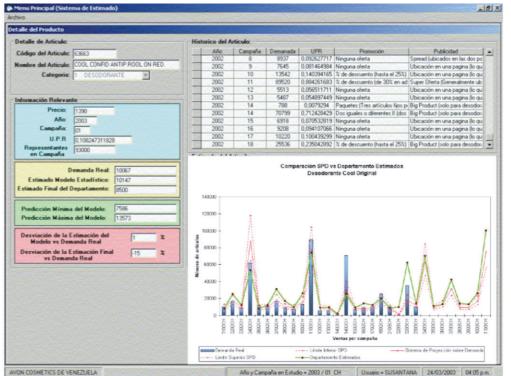
Feasibility of the automation of statistical forecast models

The development of the first and second stages of the statistical model is part of the system's programming in a way that is clear for the end user, adapting the estimates according to the new levels of tendency experienced by each product over time.

Finally, besides the statistical foundation of the modeling used, which is based on market variables, the demand forecast system presents other interesting aspects for the user:

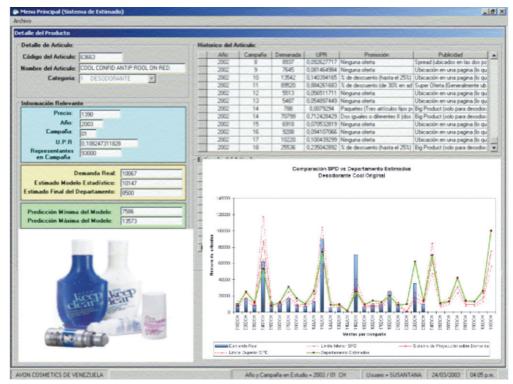
- Graphs per product: in this way, the estimator can observe graphically the product's continuity (actual demand, estimated statistical model and department estimate) of both the current and previous year.
- Based on the progress made in the digital image system, the estimator can see the appearance of the product in the future sales catalog.
- Finally, the demand forecast system (DFS) also has the option of grouping products with equivalent variables, as in the case of lipsticks, enamels, deodorants, etc.





Source: Project results.

FIGURE 11. Demand forecast system. Statistical forecast models module with product image



Source: Project results.

Results and discussion

It must always be kept in mind that it is practically impossible to make a perfect forecast. Many factors of the business context cannot be forecast with certainty. Therefore, instead of expecting a perfect forecast, it is much more important to constantly review the forecasts and to learn to live with inaccurate forecasts (Chase et al., 2005). This does not mean that we should not try to improve the forecasting model or methodology, but that we should try to find and use the most convenient method so that the forecasts are as reasonable as possible and have stable structures and procedures that can be considered within an automated forecasting tool.

In this sense, the demand forecast system is applied as a simulation tool. By introducing the causal variables, a mathematical estimate of the potential and inhibiting impacts of demand can be obtained in shorter times than if historical information was manually analyzed.

In addition to the simulation tool based on the statistical model described above, the system has a module for consulting historical and estimated information of the product's sales code, in order to facilitate the search and analysis of the variables and the product's image in the catalog, in order to optimize the final estimate of the concept at the SKU level.

The DFS automated tool was created with the aim of continuously improve the estimation process in direct or catalog sales companies thanks to the use of an application that provides solid historical information, unifies data and feeds itself from other departments, so that the end user can simulate scenarios directly from his computer by accessing all the information available. Forecasting statistical models have an added value by offering the demand estimator or planner the option of simulating scenarios mathematically, providing an additional method for subsequent evaluation.

In the case of Avon Cosmetics, the estimates have three valuation stages per product for each future catalog: preliminary stage, revision stage and final stage. During these stages, the estimator can perform a simple mathematical simulation provided by a statistical model or a more detailed assessment according to similar historical characteristics or novel aspects that can make the estimates fluctuate beyond their mathematical evaluation.

Conclusions

Establishing the conditions that can affect a sensitive aspect such as the demand of a certain product is a very complex work. It should be noted that a possible impact association is mentioned and not direct causality. However, the efforts made are worthwhile depending on the use given to the information generated by the analysis proposed. In the field of forecasts, the design phase is the most important, given that a bad decision made in this stage may bring as a consequence loss of resources, time and even planning based on wrong information. The design and adaptation of the appropriate statistical techniques are the starting point of any forecast in order to achieve the objectives previously determined.

The demand estimator or planner in direct sale companies must be an employee with wide knowledge both in the quantitative and qualitative areas and marketing, sales and supply chain. In other words, an estimator has to be a professional with knowledge in all areas of the organization, since his daily tasks have to do with aspects related to different areas of the organization.

The estimates and trends sector as an area is involved with the two main channels of a direct sales company: marketing and supply chain. An automated application guarantees the rapid assessment, simulation and evaluation of various future sales scenarios, based on historical information and statistical quantification of similar scenarios.

The estimate department is the final link in the marketing chain, since when estimating the final forecast reports are generated and these must be in line with the financial goals originally proposed by the marketing department's planning. Likewise, the estimates area is the starting point of the supply chain. Through the estimated units, the purchase area acquires the consumables, components, ingredients or finished products required, in order to guarantee a final service to the sales representative.

Due to this high level of responsibility, any over or underestimation has consequences that are relevant for both areas. When overestimating, (final estimate > actual demand), excess inventory is generated, which affects directly the supply chain and causes a high estimated forecast that is not in line with the reality of income from real demand. Also, when underestimating (final estimate < actual demand), a deficit is generated in the supply chain, which must be compensated by the chain's "reaction capacity" and the deficit will be unavailable for the representative, which affects the entire organization both in terms of service and finances. In countries with emerging economies, the reaction ability may be limited due to the macroeconomic situation, and thus a large number of "underestimated" products are inevitably depleted.

It should be recognized that most of the areas that require forecasting models have in common the changing dynamics of their activities in terms of time. In this work, this last consideration is taken into account in the adjustment structure of residuals over time, given that demand has a pattern or trend associated to time and it can change in the short, medium or long term.

Finally, an automated forecast system should not be conceived as a competitor of the demand estimator or planner, but rather as a fundamental tool that allows the user to perform real-time queries with the versatility of simulating mathematically future scenarios by using statistical models that quantify historical information through specific techniques, but such models will never be able to accurately assess the creativity of new marketing and sales business strategies.

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